



## Semester 1 Examination 2012

## Question/Answer Booklet

**MATHEMATICS 3C****Section One:  
Calculator-free**

SOLUTIONS

Name of Student: \_\_\_\_\_

**Time allowed for this section**

Reading time before commencing work: 5 minutes  
Working time for this section: 50 minutes

**Materials required/recommended for this section*****To be provided by the supervisor***

This Question/Answer Booklet  
Formula Sheet

***To be provided by the student***

Standard items: pens, pencils, pencil sharpener, eraser, correction fluid/tape, ruler,  
highlighters  
Special items: nil

**Important note to students**

No other items may be used in this section of the examination. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Section One: Calculator-free

(50 marks)

This section has six (6) questions. Answer all questions. Write your answers in the spaces provided.

Working time: 50 minutes

Question 1

(8 marks)

(a) Simplify

(i)  $\frac{x^2+5x-14}{5x^2-20} \div \frac{x^2+12x+35}{4x+20}$

(ii)  $\frac{1}{x^2+x} + \frac{2}{x^2+2x}$  (4)

$$\begin{aligned} & \frac{(x+7)(x-2)}{5(x^2-4)} \times \frac{4(x+5)}{(x+7)(x+5)} \checkmark \\ & \frac{\cancel{(x+7)}(x-2)}{5\cancel{(x-2)}(x+2)} \times \frac{4\cancel{(x+5)}}{\cancel{(x+7)}\cancel{(x+5)}} \checkmark \\ & \frac{4}{5(x+2)} \checkmark \end{aligned}$$

$$\begin{aligned} & \frac{1}{x(x+1)} + \frac{2}{x(x+2)} \\ & = \frac{1(x+2) + 2(x+1)}{x(x+1)(x+2)} \checkmark \\ & = \frac{x+2 + 2x+2}{x(x+1)(x+2)} \\ & = \frac{3x+4}{x(x+1)(x+2)} \checkmark \end{aligned}$$

(b) The functions  $f(x)$  and  $g(x)$  are defined as follows

$f(x) = x^2 - 4$  and  $g(x) = \sqrt{x-5}$

(i) Determine expressions for  $f[g(x)]$  and  $g[f(x)]$ . (2)

$$\begin{aligned} f(g(x)) &= (\sqrt{x-5})^2 - 4 \\ &= x-5-4 \\ &= x-9 \checkmark \\ g(f(x)) &= \sqrt{(x^2-4)-5} \\ &= \sqrt{x^2-9} \checkmark \end{aligned}$$

(ii) Determine the range of  $f[g(x)]$ . (1)

$R_{f(g(x))} = \{y : y \geq -4, y \in \mathbb{R}\} \checkmark$

either  $\sqrt{\text{or } x}$

(iii) Determine the domain of  $g[f(x)]$ . (1)

$D_{g(f(x))} = \{x : x \geq 3 \text{ or } x \leq -3, x \in \mathbb{R}\}$

$\checkmark$   
either  $\sqrt{\text{or } x}$

Question 2

(9 marks)

(a) Differentiate the following with respect to  $x$ .

(i)  $f(x) = \frac{-x}{x^2+1}$  (express in simplest form) (3)

$$f'(x) = \frac{(x^2+1)(-1) - (-x)(2x)}{(x^2+1)^2} \checkmark$$

$$f'(x) = \frac{-x^2 - 1 + 2x^2}{(x^2+1)^2} \checkmark$$

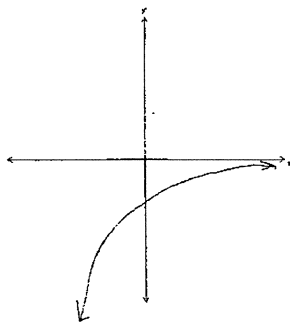
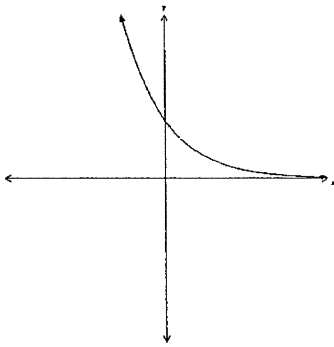
$$f'(x) = \frac{x^2 - 1}{(x^2+1)^2} \checkmark$$

(ii)  $g(x) = (2x+1)^2 e^{x^2}$  (do not simplify) (2)

$$g'(x) = (2x+1)^2 2x e^{x^2} + e^{x^2} 2(2x+1)(2) \checkmark$$

$$\text{or } 2x e^{x^2} (2x+1)^2 + 4e^{x^2} (2x+1)$$

(b) For the graph already drawn, sketch the derivative function on the axes below. (2)

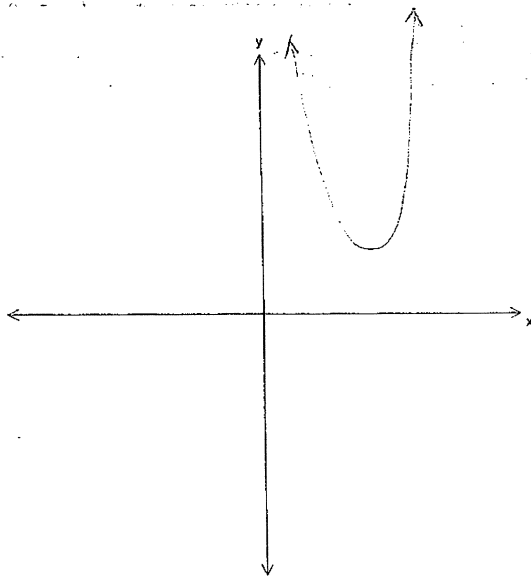
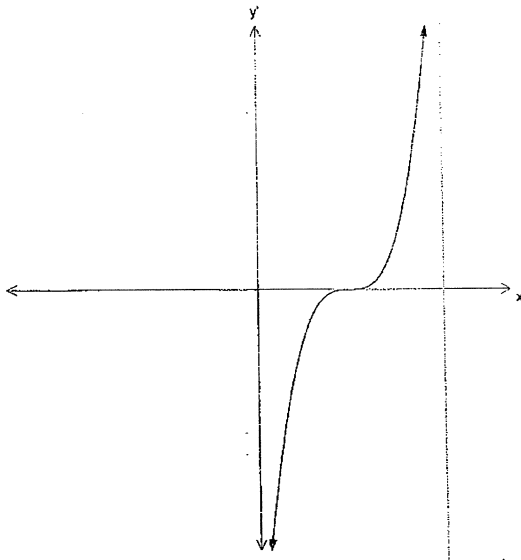


✓✓ shape all under  $x$ -axis

Question 2 (continued)

(c) Given the derivative function, sketch a possible graph of the function.

(2)



✓ Shape  
✓ Turning point

## Question 3

(7 marks)

- (a) It is claimed that the tangent line to the curve  $y = x^3 - 2x^2 - 4x + 3$  at  $x=1$  passes through the point  $(3,8)$ . Is this claim valid? Justify your answer. (5)

$$\frac{dy}{dx} = 3x^2 - 4x - 4 \quad \checkmark$$

$$\frac{dy}{dx} (1, -2) = 3 - 4 - 4 = -5 \quad \checkmark$$

Equation of tangent line at  $(1, -2)$  is  $y = -5x + c$

$$-2 = -5(1) + c$$

$$-2 = -5 + c$$

$$3 = c \quad \checkmark$$

Thus equation of tangent is  $y = -5x + 3$

Substitute  $(3, 8)$  into equation to see if it results in true statement

$$8 = -5(3) + 3$$

$$8 = -15 + 3$$

$$8 = -12 \quad (\text{false}) \quad \checkmark$$

Claim is not valid as the tangent at  $(1, -2)$  to the curve does not pass through  $(3, 8)$   $\checkmark$

- (b) Two identical biased coins are tossed together, and the outcome is recorded. After a large number of trials it is observed that the probability that both coins land showing heads is 0.36.

What is the probability that both coins land showing tails? (2)

$$P(2H) = 0.36 \Rightarrow P(1\text{ head}) = 0.6$$

$$\therefore P(1\text{ Tail}) = 0.4 \quad \checkmark$$

$$\text{Hence } P(2\text{ Tails}) = 0.4 \times 0.4 = 0.16 \quad \checkmark$$

Question 4

(8 marks)

The volume of a certain rectangular box is given by the equation  $V = x^3 - 5x^2 - 8x + 48$

(i) The height of the box is  $(4-x)$  units. (2)

Show that  $\frac{x^3 - 5x^2 - 8x + 48}{4-x}$  results in the expression  $-x^2 + x + 12$ .

$$\begin{array}{r} -x^2 + x + 12 \\ -x+4 \overline{) x^3 - 5x^2 - 8x + 48} \\ \underline{-x^3 + 4x^2} \phantom{+ 48} \\ -x^2 - 8x + 48 \\ \underline{-x^2 + 4x} \phantom{+ 48} \\ -12x + 48 \\ \underline{-12x + 48} \\ 0 \end{array}$$

alternatively

$$\begin{aligned} & (4-x)(-x^2 + x + 12) \\ &= -4x^2 + 4x + 48 + x^3 - x^2 - 12x \\ &= x^3 - 5x^2 - 8x + 48 \end{aligned}$$

✓

(ii) In the context of this question, what does  $-x^2 + x + 12$  represent? (1)

$-x^2 + x + 12$  represents the area of the base ✓

(iii) Calculate the value of  $x$  for which the volume is a maximum. (5)

$$V = x^3 - 5x^2 - 8x + 48$$

$$\frac{dV}{dx} = 3x^2 - 10x - 8 \quad \checkmark$$

$$\text{let } \frac{dV}{dx} = 0$$

$$0 = 3x^2 - 10x - 8$$

$$0 = (3x + 2)(x - 4)$$

$$x = -\frac{2}{3} \text{ or } x = 4$$

$$\frac{d^2V}{dx^2} = 6x - 10$$

$$\left\{ \begin{array}{l} \text{when } x = -\frac{2}{3}, \frac{d^2V}{dx^2} = -14 \Rightarrow \text{Max pt} \\ \text{when } x = 4, \frac{d^2V}{dx^2} = 14 \Rightarrow \text{Min pt} \end{array} \right.$$

Thus value of  $x$  when volume is a maximum is  $x = -\frac{2}{3}$  ✓

Question 5

(10 marks)

- (a) Find the global maximum and minimum values over the interval  $\frac{1}{2} \leq x \leq 2$  (5)

of the function  $y = x + \frac{1}{2x^2}$

$$y = x + \frac{x^{-2}}{2}$$

$$y' = 1 - x^{-3}$$

$$y' = 1 - \frac{1}{x^3} \quad \checkmark$$

Let  $y' = 0$  to find stationary points

$$0 = 1 - \frac{1}{x^3}$$

$$\frac{1}{x^3} = 1$$

$$x^3 = 1$$

$$\Rightarrow x = 1 \quad \checkmark$$

$$y'' = 3x^{-4} = \frac{3}{x^4}$$

$$y''(1) = 3 \Rightarrow \text{min pt} \quad \checkmark$$

Check function values at  $x = \frac{1}{2}, 1, 2$

$$\checkmark \begin{cases} f(\frac{1}{2}) = \frac{1}{2} + \frac{1}{\frac{1}{2}} = \frac{1}{2} + 2 = 2\frac{1}{2} \\ f(1) = 1 + \frac{1}{1} = 2 \\ f(2) = 2 + \frac{1}{8} = 2\frac{1}{8} \end{cases}$$

$\checkmark$   $\left\{ \begin{array}{l} \text{Thus minimum value over interval is } 2 \\ \text{Maximum value over interval is } 2\frac{1}{2} \end{array} \right.$

- (b) Events A and B are such  $P(A) = \frac{1}{2}$ ,  $P(B) = \frac{7}{12}$  and  $P(\overline{A \cup B}) = \frac{1}{4}$

- (i) Show that event A and B are NOT mutually exclusive. (3)

$$P(A \cup B) = 1 - P(\overline{A \cup B}) = 1 - \frac{1}{4} = \frac{3}{4}$$

$$P(A) + P(B) = \frac{1}{2} + \frac{7}{12} = \frac{13}{12} \quad \checkmark$$

As  $P(A \cup B) \neq P(A) + P(B)$   $\checkmark$

A & B are not mutually exclusive

alternatively

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$\checkmark P(A \cap B) = P(A) + P(B) - P(A \cup B)$$

$$= \frac{1}{2} + \frac{7}{12} - \frac{3}{4}$$

$$= \frac{6}{12} + \frac{7}{12} - \frac{9}{12}$$

$$\checkmark = \frac{4}{12} = \frac{1}{3}$$

Since  $P(A \cap B) \neq 0 \Rightarrow$  not mutually exclusive (2)

- (ii) Hence find  $P(A \cap B)$ .

$$P(A \cap B) = P(A) + P(B) - P(A \cup B)$$

$$= \frac{1}{2} + \frac{7}{12} - \frac{3}{4} \quad \checkmark$$

$$= \frac{6}{12} + \frac{7}{12} - \frac{9}{12}$$

$$= \frac{4}{12} = \frac{1}{3} \quad \checkmark$$

Question 6

(8 marks)

(a) Differentiate  $y = \sqrt{3x^2 + 4}$  by letting  $u = 3x^2 + 4$  and using the chain rule.

Show your working.

(2)

$$\begin{aligned}
 y &= u^{\frac{1}{2}} & \frac{du}{dx} &= 6x \\
 \frac{dy}{du} &= \frac{1}{2} u^{-\frac{1}{2}} \\
 \frac{dy}{dx} &= \frac{dy}{du} \times \frac{du}{dx} \\
 &= \frac{1}{2u^{\frac{1}{2}}} \times 6x \quad \checkmark \\
 &= \frac{6x}{2u^{\frac{1}{2}}} \\
 &= \frac{3x}{(3x^2 + 4)^{\frac{1}{2}}} \quad \checkmark
 \end{aligned}$$

(b) Determine  $\int 3x^3(2x^4 - 5)^8 dx$  (2)

$$\begin{aligned}
 &= \frac{3}{8} \int 8x^3(2x^4 - 5)^8 dx \quad \checkmark \\
 &= \frac{3}{8} \left[ \frac{(2x^4 - 5)^9}{9} \right] + C \\
 &= \frac{3(2x^4 - 5)^9}{72} + C \\
 &= \frac{(2x^4 - 5)^9}{24} + C \quad \checkmark
 \end{aligned}$$

(c)  $f(x)$  is defined such that  $\int_{-3}^6 f(x) dx = 24$  and  $\int_2^6 f(x) dx = 36$

Find

(i)  $\int_{-3}^2 f(x) dx$ . (1)

$$\begin{aligned}
 &= \int_{-3}^6 f(x) dx - \int_2^6 f(x) dx \\
 &= 24 - 36 \\
 &= -12 \quad \checkmark
 \end{aligned}$$



Question 6 (continued)

(c) (ii)  $\int_{-3}^2 (4f(x)+3) dx$  (2)

$$= \int_{-3}^2 4f(x) dx + \int_{-3}^2 3 dx$$

$$= 4 \int_{-3}^2 f(x) dx + \int_{-3}^2 3 dx \quad \checkmark$$

$$= 4(-12) + 3x \Big|_{-3}^2$$

$$= -48 + (6 - (-9))$$

$$= -48 + 15$$

$$= -33 \quad \checkmark$$

(iii) Sketch a possible graph of  $y=f(x)$  for  $-3 \leq x \leq 6$ . Your graph should display the relative areas of important regions but you do not need to draw this graph to scale. (1)

